Worked Solutions

Edexcel C4 Paper K

1. (*a*) $V = \frac{4}{3}\pi r^3, \ \frac{dV}{dr} = 4\pi r^2$ **(1)**

(*b*) $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

given $r = 10$ and $\frac{dr}{dt} = 0.1, \ \frac{dV}{dt} = 4\pi \times 10^2 \times 0.1$

$= 40\pi \text{ cm}^3 \text{ s}^{-1}$ **(4)**

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2. (*a*) $2x + x \cdot \frac{1}{y}\frac{dy}{dx} + \ln y + \frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(\frac{x}{y} + 1\right) = -(2x + \ln y)$

$\frac{dy}{dx} = -\frac{(2x + \ln y)}{\left(\frac{x}{y} + 1\right)}$

at $(3, 1)$ gradient $= -\left(\frac{6 + 0}{3 + 1}\right) = -\frac{3}{2}$ **(3)**

(*b*) $6x + 2x\frac{dy}{dx} + y \cdot 2 - 10y\frac{dy}{dx} + 16\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2x - 10y + 16) = -(6x + 2y)$

$\frac{dy}{dx} = 0$ when $6x + 2y = 0$

or $\quad y = -3x$

substitute $y = -3x$ into equation of curve,

$3x^2 + 2x(-3x) - 5(9x^2) + 16(-3x) = 0$

$-48x^2 - 48x = 0$

$-48x(x + 1) = 0$

$x = 0 \quad \text{or} \quad -1$

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3. (*a*) at $P \ \ y = 0$.

$\cos t = 0$

$t = \frac{\pi}{2}$

at $t = \frac{\pi}{2}, x = \frac{\pi^2}{4}$

coordinates of P are $\left(\frac{\pi^2}{4}, 0\right)$

(*b*) (i) area $= \int y \frac{dx}{dt} \ dt = \int_0^{\frac{\pi}{2}} \cos t \cdot 2t \ dt$

(ii) $A = \int_0^{\frac{\pi}{2}} 2t \frac{d}{dt}(\sin t) \ dt$ (By parts)

$= \left[2t \sin t\right]_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} \sin t \ dt$

$= \left[2t \sin t + 2\cos t\right]_0^{\frac{\pi}{2}}$

$= \pi + 0 - (0 + 2) = \pi - 2$

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4. (*a*) $f(x) = (1 - 9x^2)^{-\frac{1}{2}}$

$$= 1 + \left(-\frac{1}{2}\right)\left(-9x^2\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(-9x^2\right)^2$$

$$= 1 + \frac{9}{2}x^2 + \frac{243}{8}x^4$$ **(3)**

(*b*) valid for $|9x^2| < 1$

$$|x^2| < \frac{1}{9}, |x| < \frac{1}{3}$$ **(1)**

(*c*) (i) $\dfrac{(1+3x)^{\frac{1}{2}}}{(1-3x)^{\frac{1}{2}}} \times \dfrac{(1+3x)^{\frac{1}{2}}}{(1+3x)^{\frac{1}{2}}} = \dfrac{1+3x}{\sqrt{(1-9x^2)}}$ **(2)**

(ii) $(1+3x)\left(1 + \frac{9}{2}x^2 + \frac{243}{8}x^4\right)$

$$= 1 + \frac{9}{2}x^2 + \frac{243}{8}x^4 + 3x + \frac{27}{2}x^3 + \frac{729}{8}x^5$$

$$= 1 + 3x + \frac{9}{2}x^2 + \frac{27}{2}x^3 + \frac{243}{8}x^4 + \frac{729}{8}x^5$$ **(2)**

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5. (*a*) $A = 160, B = 50$

[N doubles as t increases by 10] **(3)**

(*b*) (i) $t = 10, m = 500\,e^{-1} = 184$ **(1)**

(ii) $300 = 500\,e^{-0.1t}$

$$\ln\frac{3}{5} = -0.1t$$

$$t = 5.1$$ **(3)**

(iii) $\dfrac{dm}{dt} = 500(-0.1)\,e^{-0.1t}$

when $t = 20$, $\dfrac{dm}{dt} = -6.77$ gram/year

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6. (*a*) (i) $\int x \ln x \; dx = \int \ln x \frac{d}{dx}\left(\frac{x^2}{2}\right) dx$ (By p

$$= \frac{x^2}{2}\ln x - \int \frac{1}{x} \cdot \frac{x^2}{2} \; dx$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$$

(ii) $\int \ln x \; dx = \int \ln x \frac{d}{dx}(x) \; dx$

$$= x\ln x - \int \frac{1}{x} \cdot x \; dx$$

$$= x \ln x - x + c$$

(*b*) let $I = \int_{1}^{-2} x\sqrt{x+3} \; dx$ let $u = x + 3$, $\frac{du}{dx} = 1$

$\therefore \quad I = \int_{4}^{1} (u-3)u^{\frac{1}{2}} \; du$ when $x = -2$ / $x = 1$,

$$= \int_{4}^{1} \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\right) du$$

$$= \left[\frac{2}{5}u^{\frac{5}{2}} - 2u^{\frac{3}{2}}\right]_4^1 = \frac{8}{5}$$

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7. (*a*) $\dfrac{dV}{dt} = 10 - \dfrac{V}{4}$

$4\dfrac{dV}{dt} = 40 - V, \ -4\dfrac{dV}{dt} = V - 40$ **(3)**

(*b*) $\displaystyle\int \frac{1}{V-40}\,dV = -\frac{1}{4}\int dt$

$\ln(V - 40) = -\dfrac{1}{4}t + c$

$V = 100, t = 0: \quad \ln 60 = 0 + c$

$\therefore \quad \ln(V - 40) = -\dfrac{1}{4}t + \ln 60$

$\ln\dfrac{(V-40)}{60} = -\dfrac{1}{4}t$

$\dfrac{V-40}{60} = e^{-\frac{1}{4}t}$

$V = 60e^{-\frac{1}{4}t} + 40$ **(7)**

(*c*) as $t \to \infty \qquad V \to 40.$ **(1)**

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8. (*a*) l and m are perpendicular $\Rightarrow \begin{pmatrix} \\ 1 \\ -2 \end{pmatrix}$

$b - 4 = 0$

$b = 4$

l and m intersect $\Rightarrow 2 = 1 + \mu a$ ..

$-1 + \lambda = -2 + \mu \times 4$ ..

$3 - 2\lambda = -5 + \mu \times 2$ ..

solving [B] and [C], $\lambda = 3, \mu = 1$

substitute in [A], $a = 1$

(*b*) point of intersection is $\begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}$

(*c*) $\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \sqrt{5}\sqrt{9}\cos\theta$, where $\theta =$ angle bet

$0 + 2 - 4 = 3\sqrt{5}\cos\theta$

$\theta = 73°$
